Tentamen: Subatomaire fysica

Thursday 25 february 1999

naam: ______________________ studentno. ______________

1. (4 pt.) Parity conservation
   a) Prove that parity conservation forbits a static electric dipole moment in the ground state of a stable nucleus. ($\vec{D} \sim \vec{r}$)

   b) Prove that a non-zero expectation value of the helicity operator λ violates parity conservation. ($\lambda = \vec{\sigma} \cdot \vec{p}/|\vec{p}|$)

2. (14 pt.) Consider a nuclear system built from 2 nucleons.
   a) Write down the possible isospin wavefunctions for the coupling of 2 nucleons to the di-nucleon system. (use the notation $|I, I_3>$ for the isospin I and 3-component I_3)

b) What is the possible spin (J) assignment for the isospin-triplet and -singlet states, respectively?

c) The ground-state of the bound di-nucleon system, the deuteron, is the $^3S_1$ state, i.e. spin J=1 and orbital angular momentum l=0, while the $^1S_0$ state is unbound. Discuss the binding of the di-proton and the di-neutron on basis of the isospin-independence of the nuclear force.

d) Determine the magnetic dipole moment μ/μ_N of the deuteron for a pure $^3S_1$ state.
($g_l = 1$; $g_s = 5.586$ for the proton; $g_l = 0$; $g_s =$ -3.826 for the neutron)
The calculated value is about 2.5% larger than the observed value. What could be the reason?

e) What value of the electric quadrupole moment would you expect for the deuteron in a pure S-state?

f) The measured value of the electric quadrupole moment of the deuteron is Q=0.00288 b. Is this "small" or "large" and how can you explain this value qualitatively?

g) Which symmetry principle determines the allowed admixture to the deuteron ground state and which state is the most likely admixture?

3. (4 pt.) Spin dependent potential.
Low-energy neutron-proton scattering data can be described approximately by representing the internucleon potential by an attractive square-well: range a=2fm, depth =35 MeV in the $^3S_1$ state and depth = 15 MeV in the $^1S_0$ state. This potential can be expressed as follows
$V(r) = A + B\ \vec{s_1} \cdot \vec{s_2}$ for $r \leq a$,
$V(r) = 0$ for $r > a$,
where $\vec{s_1}$ and $\vec{s_2}$ are the nucleon spins.
Determine the values A (in MeV) and B (in the appropriate unit.)

4. (6 pt.) Spin-orbit coupling.
a) Express $\vec{l} \cdot \vec{s}$ in terms of j, l and s. and show that the energy separation of a nuclear spin-orbit doublet is proportional to $2l+1$.

b) In the shell model the radial dependence of the nuclear density is assumed to be of the Woods-Saxon shape.
Obtain an expression for the spin-orbit potential and sketch the radial dependence for $j = l \pm \frac{1}{2}$.

5. (6 pt.) Single-particle shell model.
The single-particle levels in a Woods-Saxon potential with spin-orbit coupling are given in appendix A.
a) Argue, why the ground state and first excited states of $^{45}_{21}$Sc have spin^{Parity} $\frac{7}{2}^-$, $\frac{3}{2}^+$, and $\frac{3}{2}^-$, respectively.

b) The magnetic moment for a nucleus with spin J is given by
$\mu_J = J\ g_J\ \mu_N$ with
$g_J\ =\ g_l\ \pm \frac{g_s - g_l}{2l+1}$ and
$g_l = 1$; $g_s = 5.586$ for the proton;
$g_l = 0$; $g_s =$ -3.826 for the neutron.
Calculate the ground-state magnetic moment of $^{45}_{21}$Sc (in units of μ_N).

6. (6 pt.) Shell filling.
Since the nuclear force is short-range attractive the lowest-energy state between two nucleons is achieved if their wavefunctions overlap maximally, i.e. if their angular momenta are oriented antiparallel. This pairing force increases strongly with the value of the angular momentum l. In this light, interpret the configuration of the following nuclei using the SPSM results in appendix A:
a) the ground states of
$^{199}_{80}\mathrm{Hg}_{119}$, $^{203}_{81}\mathrm{Tl}_{122}$, $^{207}_{82}\mathrm{Pb}_{125}$, which have spin^{Parity} values
$J^P = \frac{1}{2}^-$, $\frac{1}{2}^+$, and $\frac{1}{2}^-$, respectively;

b) the low-lying levels of $^{13}_{6}\mathrm{C}$ which are in the notation J^P(excitation energy):
$\frac{1}{2}^-$(0 MeV, groundstate); $\frac{1}{2}^+$(3.09 MeV); $\frac{3}{2}^-$(3.68 MeV); $\frac{5}{2}^+$(3.85 MeV).

7. (8 pt.) γ transitions.
For the following γ transitions, name the permitted multipoles and indicate which multipole might be the most intense in the emitted radiation:
a) $\frac{9}{2}^+ \rightarrow \frac{7}{2}^+$

b) $\frac{1}{2}^- \rightarrow \frac{7}{2}^-$

c) $1^- \to 2^+$

d) $4^+ \to 2^+$

8. (6 pt.) β decay.
a) Draw the Feynman diagram for the β decay of the neutron on the quark level.

b) Sketch and motivate the general behaviour of the momentum spectrum of β particles for 3 cases:
(1) neglecting the correction due to the Coulomb field of the final nucleus;
(2) including the Coulomb correction for β^-;
(3) including the Coulomb correction for β^+.

9. (4 pt.) Parity conservation.
   The pseudoscalar $\eta(547)$ meson is observed to decay to 3-pion final states. Explain, why the decay $\eta \to \pi^+\pi^-$ and $\eta \to \pi^0\pi^0$ have never been observed.

10. (4 pt.) Baryon resonances.
    In comparison to π^+p scattering, the cross section for π^-p scattering shows additional structure. How can you explain this and what is the relative weight of the contributing amplitudes? (use appendix C)

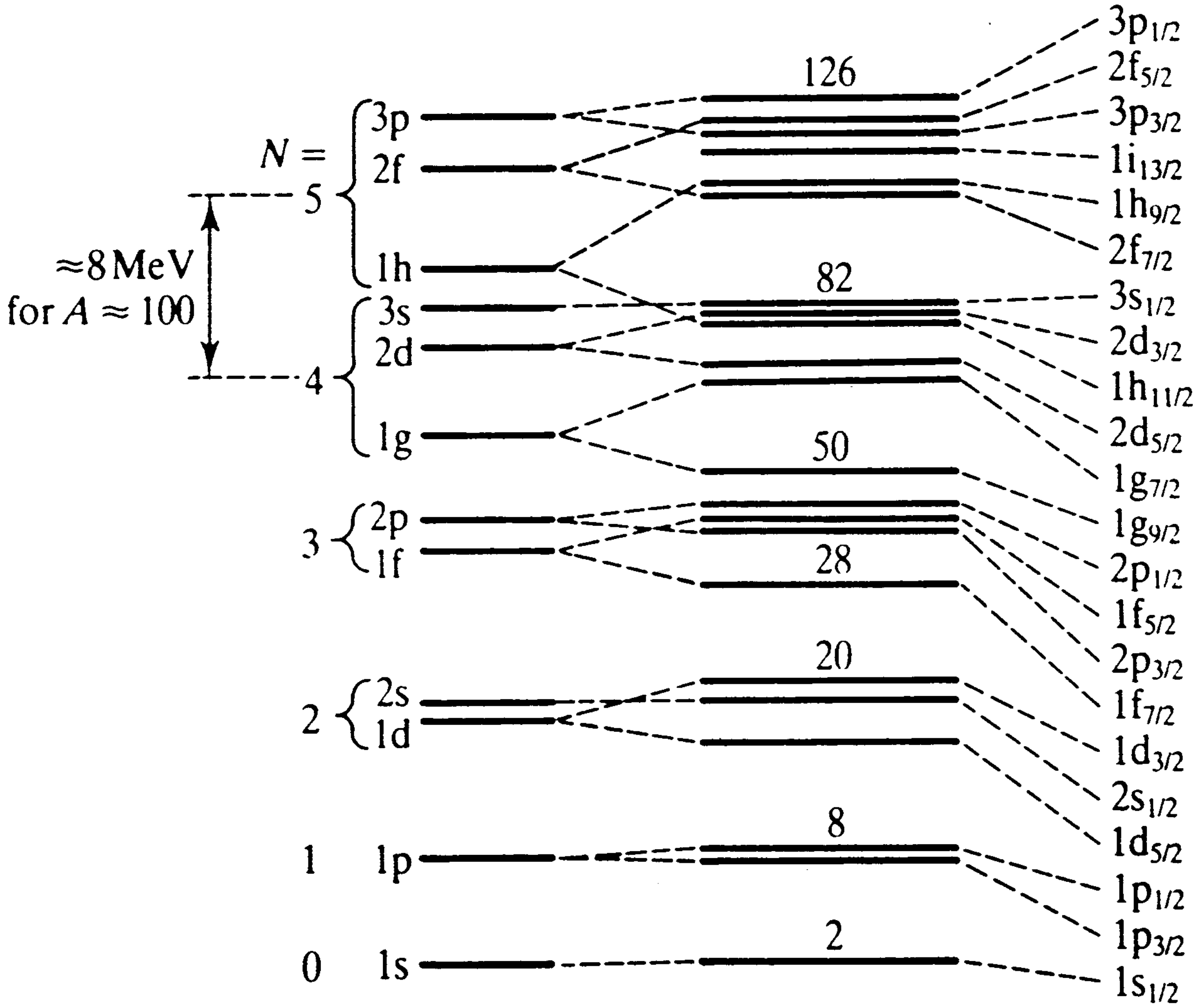
11. (6 pt.) Mass relation in U-spin multiplet.
The mass M of a particular U-spin state $|U, U_3>$ ($M = < U, U_3|H|U, U_3 >$) may be considered as a constant term m_0 equal for all members of a multiplet, a term m_s equal for all U-spin members with the same U, and a term m_v proportional to U_3; then $M = m_0 + m_s + m_v$; predict the mass relation for the neutral members of the $J^P = \frac{1}{2}^+$ baryon octet (see appendix B). The U-spin singlet state is given by $\frac{1}{2}(\Sigma^0 + \sqrt{3}\Lambda^0)$.

12. (7pt.) U-spin conservation.
The negatively charged $\frac{3}{2}^+$ baryons form a U-spin quartet, the neutral $\frac{1}{2}^+$ baryons form a U-spin triplet, and the negatively charged 0^- mesons form a U-spin doublet. Assuming U-spin conservation, determine the ratio of the decay-amplitudes for
$\Delta^- \to n\pi^-$ and $\Sigma^-(1385) \to nK^-$. (see appendix B and C).

Totaal te behalen punten:	75
Behaalde punten:	________

Cijfer: ________

Appendix A



N =
5
4
3
2
1
0
≈8 MeV for A ≈ 100
3p
2f
1h
3s
2d
1g
2p
1f
2s
1d
1p
1s
126
82
50
28
20
8
2
$3p_{1/2}$
$2f_{5/2}$
$3p_{3/2}$
$1i_{13/2}$
$1h_{9/2}$
$2f_{7/2}$
$3s_{1/2}$
$2d_{3/2}$
$1h_{11/2}$
$2d_{5/2}$
$1g_{7/2}$
$1g_{9/2}$
$2p_{1/2}$
$1f_{5/2}$
$2p_{3/2}$
$1f_{7/2}$
$1d_{3/2}$
$2s_{1/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$

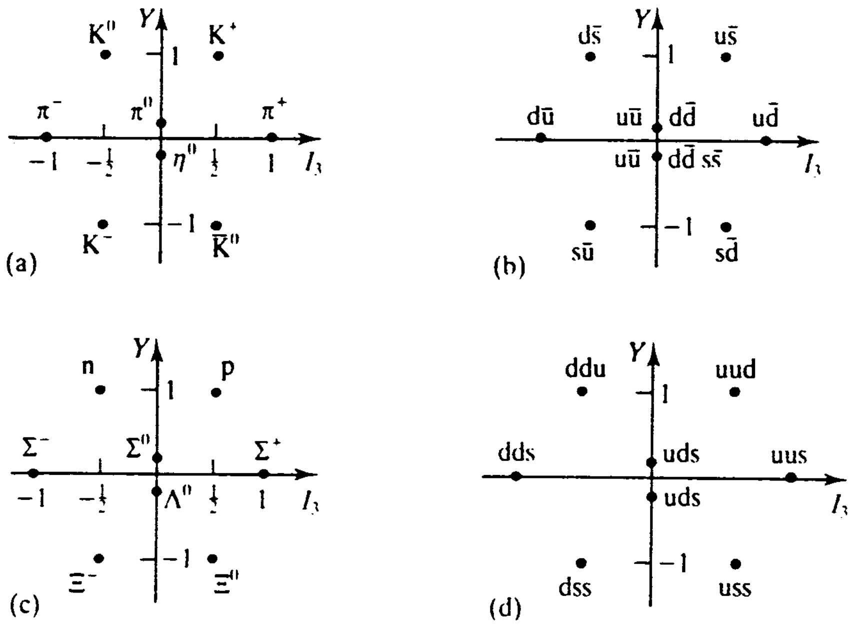


Figure 7.4
(a) The octet of 0^- mesons;
(b) quark flavour assignments for the 0^- mesons;
(c) the octet of $\frac{1}{2}^+$ baryons;
(d) quark flavour assignments for the $\frac{1}{2}^+$ baryons.

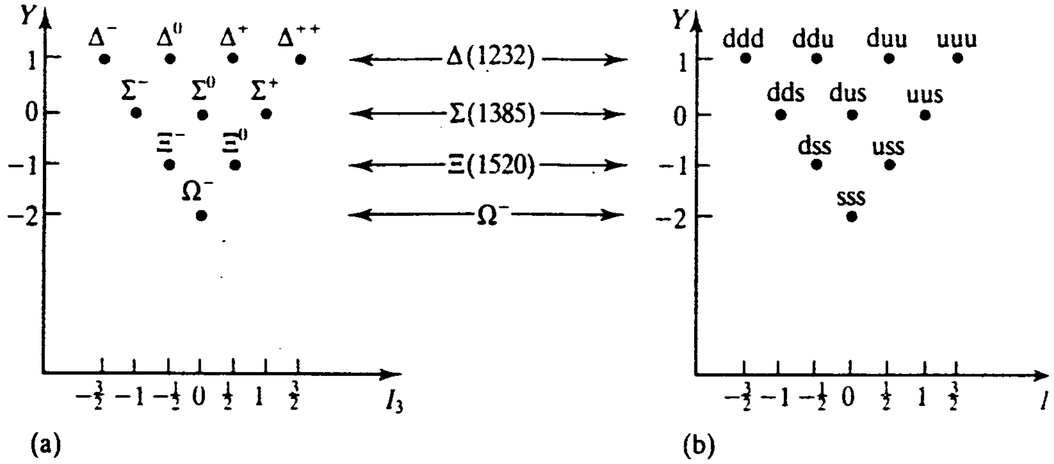


Figure 7.6
(a) The $\frac{3}{2}^+$ baryon decuplet and (b) its quark flavour content.

G *Clebsch–Gordan coefficients*

Appendix C

The vector addition or Clebsch–Gordan coefficients

$$\langle j_1 j_2 m_1 m_2 | jm \rangle = \langle jm | j_1 j_2 m_1 m_2 \rangle$$

$$j = j_1 + j_2$$

$$m = m_1 + m_2.$$

For each pair of values of j_1 and j_2 the tables are laid out in the following

(i) $j_1 = \frac{1}{2}, j_2 = \frac{1}{2}$

	j	1	1	0	1
	m	+1	0	0	−1
m_1	m_2				
$+\frac{1}{2}$	$+\frac{1}{2}$	1			
$+\frac{1}{2}$	$-\frac{1}{2}$		$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
$-\frac{1}{2}$	$+\frac{1}{2}$		$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	
$-\frac{1}{2}$	$-\frac{1}{2}$				1

(ii) $j_1 = 1, j_2 = \frac{1}{2}$

	j	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$
	m	$+\frac{3}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$
m_1	m_2						
+1	$+\frac{1}{2}$	1					
+1	$-\frac{1}{2}$		$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$			
0	$+\frac{1}{2}$		$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{3}}$			
0	$-\frac{1}{2}$				$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$	
−1	$+\frac{1}{2}$				$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{2}{3}}$	
−1	$-\frac{1}{2}$						1

(iii) $j_1 = 1, j_2 = 1$

	j	2	2	1	2	1	0	2	1	2
	m	+2	+1	+1	0	0	0	−1	−1	−2
m_1	m_2									
+1	+1	1								
+1	0		$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$						
0	+1		$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$						
+1	−1				$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$			
0	0				$\sqrt{\frac{2}{3}}$	0	$-\sqrt{\frac{1}{3}}$			
−1	+1				$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$			
0	−1							$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
−1	0							$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	
−1	−1									1